Annals of Fuzzy Mathematics and Informatics
Volume 1, No. 1, (January 2011), pp. 71- 80
ISSN 2093-9310
http://www.afmi.or.kr

Soft Γ-rings and idealistic soft Γ-rings

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Received 4 October 2010; Accepted 30 November 2010

Abstract. Soft set theory has a rich potential for applications in several directions. In this paper, we introduce soft Γ-rings, soft ideals, and idealistic soft Γ-rings. We study some of their basic properties and give several illustrative examples.

2010 AMS Classification: 16D30, 16S99, 16Y99, 03E72

Keywords: Γ-ring, Soft Γ-ring, Soft ideal, Fuzzy soft Γ-ring, Fuzzy ideal

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1. Introduction

Most of the problems in economics, engineering and environment have various uncertainties. We can not successfully use classical methods because of various uncertainties typical for those problems. There are several theories, for examples, theory of fuzzy sets [17], intuitionistic fuzzy sets [2], theory of rough sets [15], i.e., which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties as what were pointed out by Molodtsov in [11]. In 1999, Molodtsov introduced the concept of a soft set [11]. The concept of soft set is a mathematical tool for dealing with uncertainty. At present, works on the soft theory are progressing rapidly. Maji et al. [10] described the application of soft set theory to a decision making problem. Maji et al. [9] also studied several operations on the theory of soft sets. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [17].

The algebraic structure of soft set theories has been studied increasingly in recent years. Aktaş and Çağman [1] investigated basic properties of soft sets to the related concepts of fuzzy sets and rough sets. They gave a definition of soft groups and derived their basic properties. Jun et al. [7] defined soft rings, soft ideals on soft rings and idealistic soft rings. Besides, Jun [5] introduced the notion of soft BCK/BCI-algebras and soft subalgebras. Jun and Park [8] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Park et al. [14] introduced the notion of soft WS-algebras and then derived their basic properties. Feng et al. [4] initiated the study of soft semirings by using the soft set theory and investigated several related properties. Sun et al. [16] introduced a basic version of soft module theory, which extends the notion of a module by including some algebraic structures in soft sets.

In [12], N. Nobusawa introduced the notion of a Γ-ring, as more general than ring. W. E. Barnes [3] weakened slightly the conditions in the definition of the Γ-ring in the sense of Nobusawa. After these two papers are published, many mathematicians made good works on Γ-ring in the sense of Barnes and Nobusawa, which are parallel to the results in the ring theory. In [6], Jun and Lee introduced concept of fuzzy Γ-ring. After this study, many mathematicians made works in this subject. Öztürk et al. [13] discussed the fuzzy ideals in Γ-rings.

In this paper, we introduced the soft Γ-ring, soft ideal, idealistic soft Γ-ring and we study some of their basic properties and gave several examples.

2. Preliminaries

In this section we give some definitions and properties regarding soft sets [7, 9, 11] and Γ-rings [3].

Definition 2.1. [11] Let U be an initial universe set and E be a set of parameters. Let $A \subset E$. A pair (F, A) is called a soft set over U, where F is a mapping given by

$$F : A \to \wp(U)$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\eta(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [11] . These examples were also discussed in [9] and [1] .

Definition 2.2. [9] Let (η, A) and (γ, B) be two soft sets over a common universe U. The intersection of (η, A) and (γ, B) is defined to be the soft set (ν, C) satisfying the following conditions:

(i) $C = A \cap B$,

(ii) $\forall e \in C, \nu(e) = \eta(e)$ or $\gamma(e)$,(as both are same set). In this case, we write $(\eta, A) \,\widetilde{\cap}\, (\gamma, B) = (\nu, C)$.

In contrast with the above definition of soft set intersections, we alternatively define and use the following binary operation, called bi-intersection of two soft sets.

Definition 2.3. The bi-intersection of two soft sets (α, A) and (β, B) over a common universe U is defined to be the soft set (γ, C), where $C = A \cap B$ and

$\gamma : C \to \wp(U)$ is mapping given by $\gamma(x) = \alpha(x) \cap \beta(x)$ for all $x \in C$. This is denoted by $(\alpha, A) \,\tilde{\sqcap}\, (\beta, B) = (\gamma, C)$.

Definition 2.4. [9] Let (η, A) and (γ, B) be two soft sets over a common universe U. The union of (η, A) and (γ, B) is defined to be the soft set (ν, C) satisfying the following conditions:

(i) $C = A \cup B$,

(ii) for all $e \in C$,

$$\nu(e) = \begin{cases} \eta(e) & \text{if } e \in A \setminus B, \\ \gamma(e) & \text{if } e \in B \setminus A, \\ \eta(e) \cup \gamma(e) & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(\eta, A) \,\widetilde{\cup}\, (\gamma, B) = (\nu, C)$.

As a generalization of the union of two soft sets, we define the union of a nonempty family of soft sets in the following way.

Definition 2.5. [7] Let $(\alpha_i, A_i)_{i \in I}$ be a nonempty family of soft sets over a common universe U. Then the union of these soft sets is defined to be the soft set (β, B) satisfying the following conditions:

(i) $B = \cup_{i \in I} A_i$, (ii) for all $e \in B$,

$$\beta(e) = \begin{cases} F_i(e) & e \in A_i - \underset{j \in J}{\cup} \{A_j | \, j \in I - \{i\}\} \\ \underset{i \in I_0}{\cup} F_i(e) & e \in \underset{i \in I_0}{\cap} A_{i_0}, \text{ for all finite } I_0 \subset I \end{cases}$$

In this case, we write $\underset{i \in I}{\widetilde{\cup}} (\alpha_i, A_i) = (\beta, B)$.

Definition 2.6. [7] Let $(\alpha_i, A_i)_{i \in I}$ be soft sets over a common universe U. Then the intersection of these soft sets is defined to be the soft set (β, C) satisfying the following conditions:

(i) $C = \cap_{i \in I} A_i$,

(ii) For all $e \in C$, there exits an $i_0 \in I$ such that $\beta(e) = \alpha_{i_0}(e)$. In this case, we write $\underset{i \in I}{\cap} (\alpha_i, A_i) = (\beta, C)$.

Definition 2.7. [9] If (η, A) and (γ, B) be two soft sets over a common universe U, then " (η, A) AND (γ, B) " denoted by $(\eta, A) \,\widetilde{\wedge}\, (\gamma, B)$ is defined by

$(\eta, A) \,\widetilde{\wedge}\, (\gamma, B) = (\nu, A \times B)$, where $\nu(a, b) = \eta(a) \cap \gamma(b)$ for all $(a, b) \in A \times B$.

Definition 2.8. [9] If (η, A) and (γ, B) be two soft sets over a common universe U, then " (η, A) OR (γ, B) " denoted by $(\eta, A) \,\widetilde{\vee}\, (\gamma, B)$ is defined by

$(\eta, A) \,\widetilde{\vee}\, (\gamma, B) = (\nu, A \times B)$, where $\nu(a, b) = \eta(a) \cup \gamma(b)$ for all $(a, b) \in A \times B$.

Definition 2.9. [7] Let $(\alpha_i, A_i)_{i \in I}$ be soft sets over a common universe U. Then,

(1) $(\beta, B) = \underset{i \in I}{\widetilde{\wedge}} (\alpha_i, A_i)$ is a soft set such that $B = \prod_{i \in I} A_i$ and $\beta(e) = \underset{i \in I}{\cap} \alpha_i(e_i)$ for all $e = (e_i)_{i \in I} \in B$.

(2) $(\beta, C) = \underset{i \in I}{\widetilde{\vee}} (\alpha_i, A_i)$ is a soft set such that $C = \prod_{i \in I} A_i$ and $\beta(e) = \underset{i \in I}{\cup} \alpha_i(e_i)$ for all $e = (e_i)_{i \in I} \in C$.

Definition 2.10. [9] For two soft sets (η, A) and (γ, B) over a common universe U, we say that (η, A) is a soft subset of (γ, B), denoted by $(\eta, A) \,\widetilde{\subset}\, (\gamma, B)$, if it satisfies:
(i) $A \subset B$,
(ii) For every $e \in A$, $\eta(e)$ and $\gamma(e)$ are identical approximations.

Definition 2.11. [7] Let (F, A) be a soft set over R. Then (F, A) is called a soft ring over R if $F(x)$ is a subring of R for all $x \in A$.

Definition 2.12. [3] If $M = \{a, b, c, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ are additive abelian groups and for all $a, b, c \in M$ and all $\alpha, \beta \in \Gamma$, the following conditions are satisfied
(i) $a\alpha b \in M$,
(ii) $(a + b)\alpha c = a\alpha c + b\alpha c$, $a(\alpha + \beta)b = a\alpha b + a\beta b$, $a\alpha(b + c) = a\alpha b + a\alpha c$,
(iii) $(a\alpha b)\beta c = a\alpha(b\beta c)$,
then M is called a Γ-ring.

Definition 2.13. [3] Let M be a Γ-ring. A subset U of M is a left (right) ideal of M if U is an additive subgroup of M and

$$M\Gamma U = \{a\alpha u \mid a \in M,\, \alpha \in \Gamma,\, u \in U\} \quad (U\Gamma M)$$

is contained in U. If U is both a left an right ideal, then U is a two-sided ideal, or simply an ideal of M.

3. Soft Γ-rings

In what follows let M be a Γ-ring and a nonempty set and R will refer to an arbitrary ternary relation among an element of M, an element of Γ and an element of M, that is, R is a subset of $M \times \Gamma \times M$ unless otherwise specified. A set valued function $H : N \longrightarrow \wp(M)$ can be defined as $H(a) = \{b \in M \mid R(a, \alpha, b), \forall \alpha \in \Gamma\}$ for all $a \in N$. The pair (H, N) is then a soft set over M.

Definition 3.1. Let (H, N) be a soft set over Γ-ring M. Then (H, N) is called a soft Γ-ring over M if $H(a)$ is a subΓring of M, for all $a \in N \subset M$. This is denoted by (H, N, Γ).

Example 3.2. For consider the additively abelian groups
$M = \left\{ \begin{bmatrix} \bar{0} & \bar{0} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{1} & \bar{0} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{1} & \bar{1} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{0} & \bar{1} & \bar{0} \end{bmatrix} \right\} \subset (\mathbb{Z}_2)_{1\times 3}$ and
$\Gamma = \left\{ \begin{bmatrix} \bar{0} \\ \bar{0} \\ \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{0} \\ \bar{1} \\ \bar{0} \end{bmatrix} \right\} \subset (\mathbb{Z}_2)_{3\times 1}$ with addition defined as matrices addition.
Let $\cdot : M \times \Gamma \times M \longrightarrow M$, $(a, \alpha, b) \longmapsto a\alpha b$.
Therefore we have that:
(i) $a\alpha b \in M$,
(ii) $(a + b)\alpha c = a\alpha c + b\alpha c$, $a(\alpha + \beta)b = a\alpha b + a\beta b$, $a\alpha(b + c) = a\alpha b + a\alpha c$,
(iii) $(a\alpha b)\beta c = a\alpha(b\beta c)$, for all $a, b, c \in M$ and all $\alpha, \beta \in \Gamma$.
Hence M is a Γ-ring.
Let $N = \left\{ \begin{bmatrix} \bar{0} & \bar{0} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{1} & \bar{1} & \bar{0} \end{bmatrix} \right\} \subset M$ and $H : N \longrightarrow \wp(M)$ be a set valued function defined by
$H(a) = \left\{ b \in M \mid R(a, \alpha, b) \Leftrightarrow a\alpha b \in \left\{ \begin{bmatrix} \bar{0} & \bar{0} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{1} & \bar{1} & \bar{0} \end{bmatrix}, \begin{bmatrix} \bar{0} & \bar{1} & \bar{0} \end{bmatrix} \right\}, \forall \alpha \in \Gamma \right\}$
for all $a \in N$.

Obviously $H([\ \bar{0}\ \ \bar{0}\ \ \bar{0}\ ]) = \{[\ \bar{0}\ \ \bar{0}\ \ \bar{0}\ ]\}$ and
$H([\ \bar{1}\ \ \bar{1}\ \ \bar{0}\ ]) = \{[\ \bar{0}\ \ \bar{0}\ \ \bar{0}\ ], [\ \bar{1}\ \ \bar{1}\ \ \bar{0}\ ], [\ \bar{0}\ \ \bar{1}\ \ \bar{0}\ ]\}$
are subΓ-rings of M. Hence (H, N) is a soft Γ-ring of M.

Theorem 3.3. *If (F, A) and (G, B) are two soft Γ-rings over Γ-ring M, then $(F, A) \tilde{\wedge} (G, B)$ is a soft Γ-ring over Γ-ring M.*

Proof. Using Definition 2.7, we have that $(F, A) \tilde{\wedge} (G, B) = (H, A \times B)$ where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$. Since $F(x)$ and $G(y)$ are subΓ-rings of Γ-ring M the intersection $F(x) \cap G(y)$ is also subΓ-ring of Γ-ring M. Thus $H(x, y)$ is a subΓ-ring of Γ-ring M for all $(x, y) \in A \times B$ and so $(F, A) \tilde{\wedge} (G, B)$ is a soft Γ-ring over Γ-ring M. □

Definition 3.4. Let (F, A) and (G, B) are two soft Γ-rings over Γ-ring M. The soft Γ-ring (G, B) is called a soft subΓ-ring of (F, A), if it satisfies:
(i) $B \subset A$,
(ii) $G(x)$ is subΓ-ring of $F(x)$ for all $x \in B$.

Proposition 3.5. *Let (F, A) and (G, A) are two soft Γ-rings over Γ-ring M.*
(i) *$(F, A) \tilde{\cap} (G, A)$ is a soft Γ-ring over Γ-ring M.*
(ii) *If $F(x) \subset G(x)$ for all $x \in A$, then (F, A) is soft subΓ-ring of (G, A).*

Proof. (i) Using Definition 2.2, we can write $(F, A) \tilde{\cap} (G, A) = (H, A)$, where $H(x) = F(x)$ or $G(x)$ for all $x \in A$. Since (F, A) and (G, A) are soft Γ-rings over Γ-ring M, $F(x)$ and $G(x)$ are subΓ-rings of M for all $x \in A$. Thus $H(x)$ is a subΓ-ring of M for all $x \in A$, and so $(F, A) \tilde{\cap} (G, A) = (H, A)$ is a soft Γ-ring over Γ-ring M.
(ii) Straightforward. □

Definition 3.6. Let $(F_i, A_i)_{i \in I}$ be soft Γ-rings over a common Γ-ring M. Then the intersection of these soft Γ-rings is defined to be the soft Γ-ring (G, B) satisfying the following conditions:
(1) $B = \cap_{i \in I} A_i$,
(2) For all $e \in B$, there exits an $i_0 \in I$ such that $G(e) = F_{i_0}(e)$. In this case, we write $\underset{i \in I}{\cap} (F_i, A_i) = (G, B)$.

Definition 3.7. Let $(F_i, A_i)_{i \in I}$ be soft Γ-rings over a common Γ-ring M. Then,
(1) $(G, B) = \underset{i \in I}{\tilde{\wedge}} (F_i, A_i)$ is a soft Γ-ring such that
$B = \prod_{i \in I} A_i$ and $G(e) = \underset{i \in I}{\cap} F_i(e_i)$ for all $e = (e_i)_{i \in I} \in B$.
(2) $(G, C) = \underset{i \in I}{\tilde{\vee}} (F_i, A_i)$ is a soft Γ-ring such that
$C = \prod_{i \in I} A_i$ and $G(e) = \underset{i \in I}{\cup} F_i(e_i)$ for all $e = (e_i)_{i \in I} \in C$.

Theorem 3.8. *Let (F, A) be a soft set over Γ-ring M. If $\{(F_i, A_i) \mid i \in I\}$ is a nonempty family of soft subΓ-rings of (F, A) where I is an index set, then,*
(1) *$\underset{i \in I}{\cap} (F_i, A_i)$ is a soft subΓ-ring of (F, A),*

(2) $\underset{i\in I}{\wedge}(F_i, A_i)$ *is a soft sub*Γ*-ring of* (F, A),

(3) $\underset{i\in I}{\vee}(F_i, A_i)$ *is a soft sub*Γ*-ring of* (F, A), *where* $A_i \cap A_j = \varnothing$ *for all* $i, j \in I$.

Proof. (1) Using Definition 3.6 and since (F_i, A_i) soft subΓ-rings of (F, A) for all $i \in I$, we have $G : B \to \wp(M)$ by $G(x) = F_i(x)$ for all $x \in B = \underset{i\in I}{\cap} A_i \subset A$ and $i \in I$. In this case, $F_i(x)$ is a subΓ-ring of Γ-ring M for all $x \in B$ and $i \in I$ and so $\underset{i\in I}{\cap} F_i(x)$ is a subΓ-ring of Γ-ring M. Thus, (G, B) is a soft Γ-ring over Γ-ring M. Hence, $(G, B) = \underset{i\in I}{\cap}(F_i, A_i)$ is a soft subΓ-ring of (F, A) by Definition 3.6.

The proofs of (2) and (3) can be written similarly. □

Definition 3.9. Let (F, N) be a soft Γ-ring over Γ-ring M. A soft set (G, U) over Γ-ring M is called a soft left(right) ideal of (F, N), denoted by $(G, U) \mathrel{\widetilde{\lhd}_\Gamma} (F, N)$, if it satisfies:

(i) $U \subset N$,

(ii) $F(x)\Gamma G(x)$ is contained in $G(x)$ for all $x \in U$. $(G(x)\Gamma F(x))$

Example 3.10. For consider the additively abelian groups

$M = \left\{\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{1} & \bar{0} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{0} & \bar{1} & \bar{0}\end{bmatrix}\right\} \subset (\mathbb{Z}_2)_{1\times 3}$ and

$\Gamma = \left\{\begin{bmatrix}\bar{0}\\ \bar{0}\\ \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{0}\\ \bar{1}\\ \bar{0}\end{bmatrix}\right\} \subset (\mathbb{Z}_2)_{3\times 1}$ with addition defined as matrices addition.

Let $\cdot : M \times \Gamma \times M \longrightarrow M$, $(a, \alpha, b) \longmapsto a\alpha b$.

We know that M is a Γ-ring.

Let $N = \left\{\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}\right\} \subset M$ and $F : N \longrightarrow \wp(M)$ be a set valued function defined by

$F(a) = \left\{b \in M \mid R(a, \alpha, b) \Leftrightarrow a\alpha b \in \left\{\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{0} & \bar{1} & \bar{0}\end{bmatrix}\right\}, \forall\alpha \in \Gamma\right\}$
for all $a \in N$.

We know that, (F, N) is a soft Γ-ring of M from Example 3.2.

Let $U = \left\{\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}, \begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}\right\} \subseteq N$ and $G : U \longrightarrow \wp(N)$ be a set valued function defined by

$G(a) = \left\{b \in N \mid R(a, \alpha, b) \Leftrightarrow a\alpha b \in \left\{\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}\right\}, \forall\alpha \in \Gamma\right\}$ for all $a \in U$.

Obviously $G(\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}) = N$, $G(\begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}) = N$. Therefore we have that:

(i) $U \subset N$,

(ii) $F(\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix})\Gamma G(\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix})$, $G(\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix})\Gamma F(\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix})$ are contained in $G(\begin{bmatrix}\bar{0} & \bar{0} & \bar{0}\end{bmatrix}) = N$ and

$F(\begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix})\Gamma G(\begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}), G(\begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix})\Gamma F(\begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix})$ are contained in $G(\begin{bmatrix}\bar{1} & \bar{1} & \bar{0}\end{bmatrix}) = N$.

Hence soft set (G, U) over Γ-ring M is a soft ideal of (F, N), denoted by $(G, U) \mathrel{\widetilde{\lhd}_\Gamma} (F, N)$.

Theorem 3.11. *Let* (F, A) *be a soft* Γ*-ring over* Γ*-ring* M. *For any soft sets* (G_1, U_1) *and* (G_2, U_2) *over* Γ*-ring* M *where* $U_1 \cap U_2 \neq \varnothing$, *we have,*

$$(G_1, U_1) \mathrel{\widetilde{\lhd}_\Gamma} (F, A), (G_2, U_2) \mathrel{\widetilde{\lhd}_\Gamma} (F, A) \Rightarrow (G_1, U_1) \mathbin{\widetilde{\cap}} (G_2, U_2) \mathrel{\widetilde{\lhd}_\Gamma} (F, A).$$

Proof. Using Definition 2.2, we can write $(G_1, U_1) \mathbin{\widetilde{\cap}} (G_2, U_2) = (G, U)$, where

$U = U_1 \cap U_2$ and $G(x) = G_1(x)$ or $G_2(x)$ for all $x \in U$. Obviously, $U \subset A$ and $G : U \to \wp(M)$ is a mapping. Hence (G, U) is a soft set over Γ-ring M. Since $(G_1, U_1) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$ and $(G_2, U_2) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$, we know that $G(x) = G_1(x)$ is ideal of $F(x)$ or $G(x) = G_2(x)$ is ideal of $F(x)$ for all $x \in U$.

Hence $(G_1, U_1) \mathbin{\tilde{\cap}} (G_2, U_2) = (G, U) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$. This completes the proof. □

Theorem 3.12. *Let (F, A) be a soft Γ-ring over Γ-ring M. For any soft sets (G, I) and (H, J) over Γ-ring M in which I and J are disjoint, we have*

$$(G, I) \mathrel{\tilde{\lhd}_\Gamma} (F, A), (H, J) \mathrel{\tilde{\lhd}_\Gamma} (F, A) \Rightarrow (G, I) \mathbin{\tilde{\cup}} (H, J) \mathrel{\tilde{\lhd}_\Gamma} (F, A).$$

Proof. Assume that $(G, I) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$ and $(H, J) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$. By means of Definition 2.4, we can write $(G, I) \mathbin{\tilde{\cup}} (H, J) = (\Im, U)$ where $U = I \cup J$ and for every $x \in U$,

$$\Im(x) = \begin{cases} G(x) & \text{if } x \in I \setminus J, \\ H(x) & \text{if } x \in J \setminus I, \\ G(x) \cup H(x) & \text{if } x \in I \cap J. \end{cases}$$

Since $I \cap J = \varnothing$, either $x \in I \setminus J$ or $x \in J \setminus I$ for all $x \in U$. If $x \in I \setminus J$, then $\Im(x) = G(x)$ is ideal of $F(x)$ since $(G, I) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$. If $x \in J \setminus I$, then $\Im(x) = H(x)$ is ideal of $F(x)$ since $(H, J) \mathrel{\tilde{\lhd}} (F, A)$. Thus $\Im(x)$is ideal of $F(x)$ for all $x \in U$, and so $(G, I) \mathbin{\tilde{\cup}} (H, J) = (\Im, U) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$. □

If I and J are not disjoint in Theorem 3.12, then Theorem 3.12 is not true in general.

Theorem 3.13. *Let (F, A) be a soft Γ-ring over Γ-ring M. If $\{(F_j, I_j) | \, j \in J\}$ is a nonempty family of soft ideals of (F, A), where J is an index set, then,*

(1) $\underset{j \in J}{\cap} (F_j, I_j) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$,

(2) $\underset{j \in J}{\wedge} (F_j, I_j) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$,

(3) $\underset{j \in J}{\vee} (F_j, I_j) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$, *where $I_j \cap I_k = \{0\}$ for all $j, k \in I$.*

Proof. (1) Using Definition 2.6 and (F_j, I_j) are soft ideals of (F, A) for all $j \in J$, we have $H : C \to \wp(M)$ by $H(x) = F_j(x)$ for all $x \in C = \underset{j \in J}{\cap} I_j \subset A$ and for all $j \in J$. In this case, (H, C) is a soft set over Γ-ring M and also $F_j(x)$ is ideal of $F(x)$ for all $x \in C$ and for all $j \in J$ and so $\underset{j \in J}{\cap} (F_j, I_j)$ is ideal of $F(x)$ for all $x \in C$. Thus $\underset{j \in J}{\cap} (F_j, I_j) = (H, C) \mathrel{\tilde{\lhd}_\Gamma} (F, A)$ by Definition 3.9.

Using Definition 3.7(1) and Definition 3.7(2), the proofs of (2) and (3) can be written similarly. □

4. Idealistic Soft Γ-rings

Definition 4.1. Let (F, A) be a soft Γ-ring over Γ-ring M. Then (F, A) is called idealistic soft Γ-ring over Γ-ring M if $F(x)$ is an ideal of Γ-ring M for all $x \in A$.

Since every ideal of a Γ-ring is a subΓ-ring, we know that every idealistic soft Γ-ring over a Γ-ring M is a soft Γ-ring over Γ-ring M but the converse is not true in general.

Proposition 4.2. *Let (F, A) and (F, B) be soft sets over Γ-ring M where $B \subseteq A \subseteq M$. If (F, A) is an idealistic soft Γ-ring over Γ-ring M then so is (F, B).*

Proof. Straightforward. □

The converse of Proposition 4.2 is not true in general.

Theorem 4.3. *Let (F, A) and (G, B) be two idealistic soft Γ-rings over Γ-ring M. If $A \cap B \neq \varnothing$, then the intersection $(F, A) \widetilde{\cap} (G, B)$ is an idealistic soft Γ-ring over Γ-ring M.*

Proof. Using Definition 2.2, we can write $(F, A)\widetilde{\cap}(G, B) = (H, C)$, where $C = A \cap B$ and $H(x) = F(x)$ or $G(x)$ for all $x \in C$. Note that $H : C \to \wp(M)$ is a mapping, and therefore (H, C) is a soft set over Γ-ring M. Since (F, A) and (G, B) be two idealistic soft Γ-rings over Γ-ring M it follows that $H(x) = F(x)$ is an ideal of Γ-ring M or $H(x) = G(x)$ is an ideal of Γ-ring M for all $x \in C$. Hence $(H, C) = (F, A) \widetilde{\cap} (G, B)$ is an idealistic soft Γ-ring over Γ-ring M. □

Theorem 4.4. *Let (F, A) and (G, B) be two idealistic soft Γ-rings over Γ-ring M. If A and B are disjoint, then the union $(F, A) \widetilde{\cup} (G, B)$ is an idealistic soft Γ-ring over Γ-ring M.*

Proof. Using Definition 2.4, we can write $(F, A)\widetilde{\cup}(G, B) = (H, C)$, where $C = A \cup B$ and for every $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

Since $A \cap B \neq \varnothing$, either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in C$. If $x \in A \setminus B$, then $H(x) = F(x)$ is an ideal of Γ-ring M since (F, A) is an idealistic soft Γ-ring over Γ-ring M. If $x \in B \setminus A$, then $H(x) = G(x)$ is an ideal of Γ-ring M since (G, B) is an idealistic soft Γ-ring over Γ-ring M. Hence $(H, C) = (F, A) \widetilde{\cup} (G, B)$ is an idealistic soft Γ-ring over Γ-ring M. □

If A and B are not disjoint in Theorem 4.4, then Theorem 4.4 is not true in general.

Theorem 4.5. *If (F, A) and (G, B) be two idealistic soft Γ-rings over Γ-ring M then $(F, A) \widetilde{\wedge} (G, B)$ is an idealistic soft Γ-ring over Γ-ring M.*

Proof. By means of Definition 2.7, we know that

$$(F, A) \widetilde{\wedge} (G, B) = (H, A \times B),$$

where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$. Since $F(x)$ and $G(y)$ are ideals of Γ-ring M the intersection $F(x) \cap G(y)$ is also an ideal of Γ-ring M for all $(x, y) \in A \times B$, and therefore $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$ is an idealistic soft Γ-ring over Γ-ring M. □

Definition 4.6. An idealistic soft Γ-ring (F, A) over Γ-ring M is said to be trivial (resp., whole) if $F(x) = \{0\}$ (resp.,$F(x) = M$) for all $x \in A$.

Lemma 4.7. *Let $f : M \to N$ be an epimorphism of Γ-rings. If (F, A) is an idealistic soft Γ-ring over Γ-ring M then $(f(x), A)$ is an idealistic soft Γ-ring over Γ-ring N.*

Proof. For every $x \in A$, we have $f(F)(x) = f(F(x))$ is an ideal of Γ-ring N since $F(x)$ is an ideal of Γ-ring M and its onto homomorphic image is also an ideal of Γ-ring N. Hence $(f(x), A)$ is an idealistic soft Γ-ring over Γ-ring N. $\square$

Theorem 4.8. *Let $f : M \to N$ be an epimorphism of Γ-rings and let (F, A) be an idealistic soft Γ-ring over Γ-ring M.*

(i) *If $F(x) = ker(f)$ for all $x \in A$, then $(f(F), A)$ is the trivial idealistic soft Γ-ring over Γ-ring N.*

(ii) *If (F, A) is whole, then $(f(F), A)$ is the whole idealistic soft Γ-ring over Γ-ring N.*

Proof. (i) Assume that $F(x) = ker(f)$ for all $x \in A$. Then $f(F)(x) = f(F(x)) = \{0_N\}$ for $x \in A$. Hence $(f(F), A)$ is the trivial idealistic soft Γ-ring over Γ-ring N by Lemma 4.7 and Definition 4.6.

(ii) Suppose that (F, A) is whole. Then $F(x) = M$ for all $x \in A$, and so $f(F)(x) = f(F(x)) = f(M) = N$ for all $x \in A$. It follows Lemma 4.7 and Definition 4.6 that $(f(F), A)$ is the whole idealistic soft Γ-ring over Γ-ring N. $\square$

Definition 4.9. Let (F, A) and (G, B) be two soft Γ-rings over rings M and N, respectively. Let $f : M \to N$ and $g : A \to B$ be two functions. Then the pair (f, g) is called a soft Γ-ring homomorphism if it satisfies the following conditions:

(i) f is an onto ring homomorphism,
(ii) g is onto,
(iii) $f(F(x)) = G(g(x))$ for all $\forall x \in A$.

If there exists a soft Γ-ring homomorphism between (F, A) and (G, B), we say that (F, A) is soft homomorphic to (G, B), and is denoted by $(F, A) \sim_\Gamma (G, B)$. Moreover, if f is an isomorphism and (F, A) is soft isomorphic to (G, B), which is denoted by $(F, A) \simeq_\Gamma (G, B)$.

Acknowledgements. The authors are grateful for financial support from the Research Fund of Adiyaman University under grant no. FBEBAP2010/0002

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